### Research Retreat and ARA, Sept 1-6, 2014, Japan

Location: Fujinosato at Gotemba city of Shizuoka prefecture, Japan (near Mt. Fuji)

Contact:

ARA web site: http://kenshi.miyabe.name/ara2014/

Organized by Kenshi Miyabe and Andre Nies

#### There will be 16 participants:

Laurent Bienvenu, CNRS & Université Paris 7 Joerg Brendle, Kobe University Adam Day, Victoria University of Wellington Noam Greenberg, Victoria University of Wellington Kojiro Higuchi, Chiba University Rupert Hölzl, National University of Singapore Akitoshi Kawamura, University of Tokyo Takayuki Kihara, Japan Advanced Institute of Science and Technology Rutger Kuyper, Radboud University Nijmengen Joseph S. Miller, University of Wisconsin - Madison Benoit Monin, Université Paris 7 Kenshi Miyabe, Meiji University Satyadev Nandakumar, Indian Institute of Technology Kanpur André Nies, University of Auckland Selwyn Ng, Nanyang Technological University Toru Takisaka, Kyoto University

# Programme

# Aug 31

Some arrive at Fujinosato

# Sept 1

Research and discussion

# Sept 2

10:00 **Effective analysis** (Nies with Miyabe, Hoelzl, Miller) We discuss various connections of randomness with Lebesgue density, and porosity.

## 8:00pm Workshop on new research media

We will discuss:

(a) new technologies for remote collaboration

(b) communication and research tools provided by the web, such as blogs and polymath.

# Sept 3

10:00 Algorithmic randomness and K-triviality (Miller with Kuyper, Higuchi, others)

Lowness for dimension and other topics

14:00 **Connections of randomness and set theory** (Monin with Brendle, Greenberg, Nies) Higher randomness, cardinal characteristics, and other topics

## 8:00pm Workshop on getting into good journals and conferences

# Sept 4 ARA Day 1 10AM

### **Coarse degrees and randomness**

Rutger Kuyper, Radboud University Nijmengen

Recently, several researchers have looked at the notions of coarse and generic computability introduced by Jockusch and Schupp. Informally, these two notions formalise what it means for a set to be `almost' computable. More specifically, a set X is coarsely computable if it has a computable coarse description, where a coarse description is a set D such that the symmetric difference of X and D has density 0. There is a natural reducibility notion which corresponds to this: we say that X coarsely reduces to Y if every coarse description of Y uniformly computes a coarse description of X.

We will look at the coarse degrees of random sets. Intuitively, the coarse degree of a random set should not be able to compute much, not even non-uniformly. We make this precise and show that the coarse degree of a random can only compute K-trivials at most, while we show that there are 1-randoms such that their coarse degrees do indeed compute a non-computable set. On the other hand, we show that not all K-trivials appear in this way.

This is joint work with Hirschfeldt, Jockusch and Schupp.

#### Density and the Gamma question

#### Rupert Hoelzl, NUS

We introduce and discuss an open question raised by Andrews, Cai, Diamondstone, Jockusch, and Lempp in their paper "Asymptotic density, computable traceability, and 1-randomness". First we recall the definitions of coarse and generic computation. These notions are defined for sets, but can be extended to Turing downward cones as suggested in the above article. This does not give an interesting notion for generic computation (due to a result by Bienvenu, Day, and Hölzl).

But it does for coarse computation: One can define a function Gamma which measures the lowest percentage of coarse computability of all sets in the lower cone of a Turing degree. Andrews, Cai, Diamondstone, Jockusch, and Lempp have studied this function and shown that it takes the values 0, 1/2, and 1; and no values between 1/2 and 1. The open question remains whether it can take values between 0 and 1/2. The question was presented by Downey at the CCR conference in Singapore in 2014, but since then several attempts of constructing a degree with 0< Gamma < 1/2 have failed.

We first discuss the two central proofs in the above paper, to get an idea of the type of arguments involved, and then a few vague ideas of how to approach the open question.

Coffee

## Computability aspects of recurrence theorems

Adam Day, Victoria University of Wellington

I will present some results that come from analysing the computability of recurrence theorems in topological dynamics. I will discuss some related open questions, and directions for future research in the context of ergodic theory. Via the Furstenberg correspondence principle they relate to questions in combinatorial number theory.

Lunch

2pm

### Algorithmic identification of probabilities is hard

Laurent Bienvenu, CNRS & Universite Paris 7

Suppose an infinite binary sequence is random for a Bernoulli measure with parameter p. By the law of large numbers, the frequency of zeros in the sequence tends to p, and thus we can get better and better approximations of p as we read the sequence.

We consider a similar scenario from the viewpoint of inductive inference. We now suppose p is a computable real, but also ask to learn more information on p: as we are reading more and more bits of our random sequence, we have to eventually guess the exact parameter p in the form of a Turing program. Can one do this in a uniform way for all sequences that are random for computable Bernoulli measures, or even on a sufficiently large fraction of them?

We will give a negative answer to this question. In fact, we prove a very general negative result which extends far beyond the class of Bernoulli measures. The corresponding paper is here: <u>http://arxiv.org/abs/1405.5139</u>

This is joint work with Benoit Monin and Alexander Shen.

Coffee

#### Measure-predictable scan rules

Kojiro Higuchi, Chiba University

It is a major open problem whether Martin-Lof Randomness (MLR for short) coincides with the apparently weaker notion of Kolmogorov-Loveland Randomness (KLR for short).

We talk about a recent result which may help to solve the problem negatively.

As a corollary, our result implies a theorem of Kastermans and Lempp (2010) saying that MLR is different from injective randomness, a weaker variant of KLR.

Our theorem states that MLR is still different from a variant of KLR defined via restricting the set of all scan rules to any nice set of measure- predictable scan rules.

Here we say that a scan rule s is *measure-predictable* if the set  $A^s_n$  of infinite binary sequences whose n-th value is scanned has a computable uniform measure for each natural number n. A set of predictable scan rules is *nice* if for all scan rules  $s_0,...,s_m$  in the set, there exists an infinite set B of natural numbers such that the measure of  $A^{s_k}_n$  is computable uniformly in (k,n) on the set  $\{0, cdots, m\}$ \*B.

#### **Higher randomness**

Noam Greenberg, Victoria University of Wellington

I will discuss two generalisations of Turing reducibility in higher computability.

# Sept 5 ARA Day 2

## 10am

**Highness properties of oracles and cardinal invariants of the continuum** Joerg Brendle, Kobe University

We discuss an analogy between highness properties of oracles and cardinal invariants of the continuum. We will particularly focus on highness properties corresponding to invariants of the null ideal, the meager ideal, and the sigma ideal generated by closed null sets.

Coffee

### Unfolded forcing constructions via Kreisel compactness

Takayuki Kihara, JAIST

We provide rudimentary versions of typical hyperarithmetical arboreal forcing constructions, and discuss some related open problems concerning higher randomness theory.

Lunch

# 1pm

### Normal Numbers and Transcendence

Satyadev Nandakumar, Indian Institute of Technology, Kanpur

Recall that a Liouville number z has the property that for each natural n, there exist natural numbers p,q such that z is less than  $q^-n$  away from p/q. Diophantine approximation is one of the techniques of establishing transcendence, starting with Liouville's Theorem in the 1840s that Liouville numbers are transcendental.

In this talk, we will discuss recent constructions of transcendental numbers which are also normal (Kano 1993, Nandakumar and Vangepalli 2013, Bugeaud 2002, and Becher, Heiber and Slaman 2014). We give an overview of

- the construction of Liouville numbers which are known to be normal in a given base but not necessarily in others,
- absolutely Liouville normal numbers (by Becher, Heiber and Slaman 2014)
- non-Liouville transcendental normal numbers.

We will be discussing the transcendence and the normality proofs for these numbers. We will further consider a standard way of extending these constructions to transcendental numbers of any finite-state dimension.

### Coffee

### **Bases for LR-reducibility**

Selwyn Ng, NTU

The study of algorithmic randomness, in particular of lowness properties for randomness, has led to many exciting recent developments in the area. In particular this has provided us a tool to compare the amount of inherent "information" present in sets of natural numbers, using what is known as "weak reducibilities".

In this talk we will mention some related results, and explore the notion of an LR-base for randomness. We present several new results about this class, and investigate its position in the hierarchy of jump-traceability.

This is joint work with Johanna Franklin and Reed Solomon.

# Sept 6

Everyone needs to leave Fujinosato in the morning. Some go to excursion.



from LOVE HINA.

Ara is a Japanese word standing for a weak surprise.
But "Ara Ara" in manga has a special meaning.
If some girl in manga says "Ara Ara", this means that
the girl always does a strange act which induce laugh (ten-nen in Japanese)
sometimes keen
usually senior, slow-tempo, polite, long black hair, perfect figure
Such a girl is called an "Ara Ara type" (ara-ara-kei in Japanese).