Fourth Workshop on Game-Theoretic Probability and Related Topics

Hongo Campus, The University of Tokyo, Japan

12 - 14 Nov 2012

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Contents

1 General information

GTP2012:
Fourth Workshop on Game-Theoretic Probability and Related Topics

Date:
12-14 Nov 2012

Venue:
Room 367, 373 (Seminar Room A,D), No. 6 BLDG, Faculty of Engineering, Hongo Campus, The University of Tokyo, Japan

Local Organizers:
Akimichi Takemura (takemura@stat.t.u-tokyo.ac.jp),
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This workshop is jointly sponsored by "First, Aihara Innovative Mathematical Modelling Project" and "JSPS Kakenhi 22240029".

2 Participants

Invited speakers

- Laurent Bienvenu (CNRS & University Paris 7, France)
- Gert de Cooman (Ghent University, Belgium)
- Bjørn Kjos-Hanssen (The University of Hawaii, US)
- Alexander Outkin (Sandia National Laboratories, US)
- Glenn Shafer (The University of London, UK)
- Kazuyuki Tanaka (Tohoku University, Japan)
- Vladmir Vovk (The University of London, UK)
- Vladimir V'yugin (Russian Academy of Sciences, Russia)

Participants

- Jasper De Bock (Ghent University, Belgium)
- Takayuki Kihara (JAIST, Japan)
- Masayuki Kumon (Japanese Association for Promoting Quality Assurance in Statistics, Japan)
- Joe Suzuki (Osaka University, Japan)
- Kohtaro Tadaki (Chuo University, Japan)
- Shin-ichiro Takazawa (Kobe University, Japan)
- Akimichi Takemura (The University of Tokyo, Japan)
- Kei Takeuchi (Japan)
- Ryota Matsuo (Nagoya University, Japan)

- Kenshi Miyabe (Kyoto University, Japan)
- Keita Yokoyama (Tokyo Institute of Technology, Japan)

3 Programme

Monday 12

9:00 – 9:05 Opening

Session 1. Game-Theoretic Probability. (chair: A. Takemura)

Speaker 1. 9:10 – 10:00 Glenn Shafer (The University of London, UK)
Basics of game-theoretic probability and its interpretation.

Speaker 2. 10:15 – 10:45 Shin-ichiro Takazawa (Kobe University, Japan)
The rate of convergence of strong law of large numbers and convergence of series of moderate and small deviations in the unbounded forecasting game.

Speaker 3. 11:00 – 11:30 Kenshi Miyabe (Kyoto University, Japan)
The law of the iterated logarithm in game-theoretic probability.

Speaker 4. 11:45 – 12:30 Takemura Akimichi (The University of Tokyo, Japan)
Bayesian logistic betting strategy against probability forecasting.

12:30 – 13:45 *Lunch Break.*

Session 2. Applications of Game-Theoretic Probability. (chair: G. Shafer)

Speaker 5. 13:45 – 14:45 Vladimir Vovk (The University of London, UK)
Insuring against loss of evidence and capital: review.

Speaker 6. 15:00 – 15:30 Masayuki Kumon (Japanese Association for Promoting Quality Assurance in Statistics, Japan)
Rate distortion function in betting game system.

Speaker 7. 15:45 – 16:45 Alexander Outkin (Sandia National Laboratories, US)
On Applications of Hybrid Causal-Learning Systems to Complex Systems and Markets Modeling.

Tuesday 13

Session 3. Algorithmic randomness. (chair: T. Kihara)

Speaker 8. 9:00 – 9:45 Kenshi Miyabe (Kyoto University, Japan)
An introduction to algorithmic randomness.

Speaker 9. 10:00 – 10:30 Takayuki Kihara (JAIST, Japan)
Strong nullness and lowness for randomness.

Speaker 10. 10:45 – 11:45 Laurent Bienvenu (CNRS & University Paris 7, France)
How powerful are integer-valued martingales?

Speaker 11. 12:00 – 12:30 Keita Yokoyama (Tokyo Institute of Technology, Japan)
Randomness and arithmetic.

12:30 – 13:45 *Lunch Break.*

Session 4. Universal algorithm. (chair: A. Takemura)

Speaker 12. 13:45 – 14:30 Joe Suzuki (Osaka University, Japan)
Universal Prediction without assuming either Discrete or Continuous.

Speaker 13. 14:45 – 15:45 Vladimir V'yugin (Russian Academy of Sciences, Russia)
Universal Algorithm for Online Trading Based on the Method of Calibration.

Session 5. Imprecise probability. (chair: V. Vovk)

Speaker 14. 16:00 – 17:00 Gert de Cooman (Ghent University, Belgium)
A link between Game-Theoretic Probability and Imprecise Probabilities.

Speaker 15. 17:15 – 18:00 Jasper De Bock (Ghent University, Belgium)
Imprecise multinomial processes: an overview of different approaches and how they are related to each other.

19:00 – *Dinner party.*

Wednesday 14

Session 6. Algorithmic randomness. (chair: K. Miyabe)

Speaker 16. 9:00 – 10:00 Kohtaro Tadaki (Chuo University, Japan)
The Random Oracle Methodology and Algorithmic Randomness.

Speaker 17. 10:15 – 11:15 Bjørn Kjos-Hanssen (The University of Hawaii, US)
Algorithmic randomness, Brownian motion, and stochastic calculus.

Session 7. Reverse Mathematics. (chair: K. Yokoyama)

Speaker 18. 11:30 – 12:30 Kazuyuki Tanaka (Tohoku University, Japan)
Infinite Games and Reverse Mathematics.

12:30 – 13:45 *Lunch Break.*

Session 8. Optimal Strategy. (chair: A. Takemura)

Speaker 19. 13:45 – 14:30 Kei Takeuchi (Japan)
Bayes optimal strategy for single item betting game.

Session 9. Game-theory and probability (chair: K. Miyabe)

Speaker 20. 14:45 – 15:15 Ryota Matsuo (Nagoya University, Japan)
Kuhn's theorem for natural strategies.

4 Abstracts

Basics of game-theoretic probability and its interpretation

Glenn Shafer
University of London, UK.

In the thoroughly game-theoretic approach to probability that has been explored by this community, probabilities derive from betting offers, probabilistic assertions are hypotheses are tested by implementing betting strategies, and theorems in probability are proven by constructing betting strategies. Betting offers may determine less than a probability distribution; then we have upper and lower probabilities. This game-theoretic approach can be traced back to Pascal and can be contrasted with Fermat's combinatorial approach. Game-theoretic probabilities can be objective or subjective.

The rate of convergence of strong law of large numbers and convergence of series of moderate and small deviations in the unbounded forecasting game

Shin-ichiro Takazawa.
Kobe University, Japan.

We consider the convergence rate of strong law of large numbers in the unbounded forecasting game. We show that the series of moderate deviation probabilities converges if Reality violates the law of the single logarithm. Similarly, we show that the series of small deviation probabilities converges if Reality violates the law of the iterated logarithm. Also, we give related results for a sequence of martingale differences.

The law of the iterated logarithm in game-theoretic probability

Kenshi Miyabe.
Kyoto University, Japan.

The Kolmogorov law of the iterated logarithm (LIL) (1929) provides the exact speed of the convergence of the sum of independent random variables under a condition. Subsequently, Hartman and Wintner (1941) showed that, in the case of i.i.d. random variables, the existence of a second moment is sufficient for the LIL. Shafer and Vovk (2001) studied the Kolmogorov LIL in game-theoretic probability and asked the treatment of the Hartman and Winter LIL in game-theoretic probability.

I present a new sufficient condition for the LIL in game-theoretic probability, which has a similar form to the Hartman and Winter LIL. The main idea is to add a little stronger hedges. This is the joint work with Akimich Takemura.

Bayesian logistic betting strategy against probability forecasting

Akimichi Takemura.
The University of Tokyo, Japan.

We propose a betting strategy based on Bayesian logistic regression modeling for the probability forecasting game in the framework of game-theoretic probability by Shafer and Vovk (2001). We prove some results concerning the strong law of large numbers in the probability forecasting game with side information based on our strategy. We also apply our strategy for assessing the quality of probability forecasting by the Japan Meteorological Agency. We find that our strategy beats the agency by exploiting its tendency of avoiding clear-cut forecasts.

(joint with Masayuki Kumon, Jing Li and Kei Takeuchi)

Insuring against loss of evidence and capital: review

Vladimir Vovk
The University of London, UK.

In game-theoretic probability, statistical testing can be framed as a repetitive game between two players, Forecaster and Sceptic. On each round, Forecaster sets prices for various gambles, and Sceptic chooses which gambles to make. If Sceptic multiplies by a large factor the capital he puts at risk, he has evidence against Forecaster's ability. His capital at the end of each round is a measure of his evidence against Forecaster so far. This can go up and then back down, and reporting the maximum so far instead of the current value exaggerates the evidence against Forecaster. However, the exaggeration is not gross: Sceptic's strategy can be modified in such a way that actual evidence becomes almost as strong as the exaggerated evidence. This gives a method for insuring against loss of evidence. In the context of an actual market, this becomes a method of insuring against the loss of what an investor has gained so far. There are close connections with both p-values (the central notion of classical hypotheses testing) and Bayes factors (its Bayesian counterpart).

Rate distortion function in betting game system

Masayuki Kumon.
Japanese Association for Promoting Quality Assurance in Statistics, Japan.

Among various aspects of game theoretic probability, when exploring mathematical structure of the optimal strategies in betting games, Kullback-Leibler divergence is naturally derived as the optimal exponential growth rate of the betting capital process. This structure is closely related to Kelly's strategy, which was inspired by Shannon's information theory. Suggested by these facts, the following are addressed in this talk :

- Game mutual information which measures information transmission between betting games is introduced.

- Two characteristics Game channel capacity and Game rate distortion function are defined from the mutual information, and these meanings are explained.
- The effect of the optimal strategy in conditional betting game is verified by using real stock price data.
- As an application of Game rate distortion function, an efficient lossy source coding scheme based on the optimal conditional betting strategy is proposed.

On Applications of Hybrid Causal-Learning Systems to Complex Systems and Markets Modeling

Alexander Outkin.

Agent-based models of financial markets allow a realistic representation of market dynamics arising as a result of strategic interaction of market makers and investors as intermediated by the market rules and infrastructures. However, agent-based models of complex systems in general and markets in particular are extremely difficult to calibrate, partially because much of pertinent information on strategies of the participants as well as on details of transactions, such as the identities of counter- parties to a trade, are not completely observable.

We present a conceptual and mathematical framework and software implementation that incorporates the predictive power of agent-based models within an overarching learning framework without a need for completely calibrating the agent-based models. Specifically, we use the agent-based models to represent the inner system interactions, such as making trading decisions or orders processing as a causal model; and use learning algorithms to calibrate the causal model parameters and generate prediction error estimates. The key feature of this approach is the action/observation/ learning loop: make predictions using causal models, observe actual outcomes, and improve causal models.

We demonstrate the effectiveness of this approach on an example of a market model as applied to actual markets data, and show that incompletely calibrated agent-based models can be used for prediction within a self-correcting learning framework. We further explore extensions of this approach to online learning and to real-time applications and discuss its connections to game-theoretic probability.

An introduction to algorithmic randomness

Kenshi Miyabe.
Kyoto University, Japan.

I give an introduction to algorithmic randomness, focusing on the difference from game-theoretic probability.

Strong nullness and lowness for randomness

Takayuki Kihara
JAIST, Japan.

In the theory of algorithmic randomness, there is a well-known equivalence among the data compression ratio, the Hausdorff dimension, and the growth rate of gambler's capital in the context of martingale. In this talk, we consider a set-theoretic notion whose miniaturization is expected to play an important role in the theory of algorithmic randomness. The notion is known as strong nullness which is introduced by Emil Borel in 1919 and is studied intensively in modern set theory of the real line. We can see that this notion is also related to the data compressibility (the Kolmogorov complexity), and the growth speed of gambler's capital in gambling games. Moreover, we clarify a relationship between strong nullness and lowness for randomness (a property concerning derandomization power)

How powerful are integer-valued martingales?

Laurent Bienvenu.
CNRS & University Paris 7, France.

In algorithmic randomness, one of the classical ways to define random sequences is via the so-called unpredictability paradigm: a sequence X of bits is random if no computable gambling strategy can win arbitrarily large amounts of money by betting on the bits of X. In most of the literature on the subject, the gambling strategies considered are allowed to bet arbitrarily small amounts of money. This assumption, although it leads to very useful notions of randomness (such as computable randomness, or Kolmogorov-Loveland randomness), is not very realistic in practice. Indeed, money in the "real world" is discrete: in any actual gambling situation, the amounts of money we are allowed to bet are all multiples of some minimal betting amount (e.g. one cent). This leads to the natural idea of defining randomness via discrete betting strategies. In this talk, we show that this apparently small restriction on the class of strategies considered changes dramatically the properties of random sequences.

Randomness and arithmetic

Keita Yokoyama
Tokyo Institute of Technology, Japan.

It is well-known that the notion of Martin-Löf randomness can characterize the system WWKL_0, a subsystem of second-order arithmetic which plays a key role in reverse mathematics for measure theory. Similarly, the notion of complex can characterize the system DNR_0. In this talk, I will consider the relation between randomness and arithmetic, give some characterization of randomness from the view point of arithmetic, and vice versa.

Universal Prediction without assuming either Discrete or Continuous

Joe Suzuki.
Osaka University, Japan.

In 1975, the late Prof. Tom Cover raised a well-known problem on on-line prediction (the proposition was solved negatively). Then, a rather weak problem was solved by Bailey. In this paper, we propose a universal measure to solve the problem not just for the discrete stational ergodic sequences but also for the continuous ones. We consider aslo another Cover's problem on on-line prediction which was solved by Ornstein and Algoet.

Universal Algorithm for Online Trading Based on the Method of Calibration

Vladimir V'yugin.
Russian Academy of Sciences, Russia.

We present a universal algorithm for online trading in Stock Market which performs asymptotically at least as good as any stationary trading strategy that computes the investment at each step using a fixed function of the side information that belongs to a given RKHS (Reproducing Kernel Hilbert Space). Using a universal kernel, we extend this result for any continuous stationary strategy. In this learning process, a trader rationally chooses his gambles using predictions made by a randomized well-calibrated algorithm. Our strategy is based on Dawid's notion of calibration with more general checking rules and on some modification of Kakade and Foster's randomized rounding algorithm-for computing the well-calibrated forecasts. We combine the method of randomized calibration with Vovk's method of defensive forecasting in RKHS. Unlike the statistical theory, no stochastic assumptions are made about the stock prices. Our empirical results on historical markets provide strong evidence that this type of technical trading can "beat the market" if transaction costs are ignored.

A link between Game-Theoretic Probability and Imprecise Probabilities

Gert de Cooman.
Ghent University, Belgium.

In game-theoretic probability (GTP) there is a fundamental formula (which we will call the Shafer-Vovk-Ville, or SVV, formula) for expressing lower and upper prices for a variable defined on the terminal situations of an event tree associated with a game. In GTP it is used as a given: a starting point for much of the development of the theory. In earlier work, we have shown how, for event trees that are bounded, this formula can be derived on a behavioral approach and with a different interpretation, in the context of the theory of imprecise probabilities (IP), from two rationality requirements: coherence and cut conglomerability. In the present talk, we discuss how something similar, but more involved, can also be done for unbounded event trees: besides coherence, we impose two additional rationality axioms, bounded cut conglomerability and bounded cut continuity. Interestingly, our approach shows that in deriving the SVV formula, two types of infinity have a par! t, and can be treated separately: bounded cut conglomerability tries to cope with infinity in the width of the event tree, and bounded cut continuity with infinity in its depth. These additional requirements only need to be invoked when going from local to global modes: it concerns the global uncertainty models in the tree—the uncertainty about paths—, whereas for the local models—the uncertainty about the next move in a situation—we only need to impose coherence, nothing more. We explore a number of aspects and consequences of this connection between GTP and IP.

(joint work with Jasper De Bock and Enrique Miranda)

Imprecise multinomial processes: an overview of different approaches and how they are related to each other

Jasper De Bock.
Ghent University, Belgium.

In this overview, we present and compare four different approaches to imprecise multinomial processes, which are generalizations of the classical multinomial process to the field of imprecise probability theory. Within this field, one can choose between a number of different mathematical frameworks. Amongst the most important ones, we have credal sets, coherent lower previsions and coherent sets of desirable gambles. We show how each of them can be used to model beliefs about the outcome of a single experiment. We give an overview of different ways of extending these local models to describe an infinite sequence of experiments, leading to four different types of imprecise multinomial processes. We investigate their properties, discuss the assumptions that underly them and show how they can be related to one another by imposing additional requirements. In particular, it turns out that by additionally imposing exchangeability, all four types of imprecise multinomial processes coincide, which ultimately provides us with a behavioural justification for applying sensitivity analysis to classical multinomial processes.

The Random Oracle Methodology and Algorithmic Randomness

Kohtaro Tadaki.

Chuo University, Japan.

In modern cryptography, the random oracle methodology is widely used to design and validate cryptographic schemes. It proceeds in the following two steps: First, a scheme is designed and proven secure in the random oracle model. Secondly, the random oracle is instantiated with a cryptographic hash function. At present, however, there is no theoretical justification for the second step. We investigate this problem of instantiation, based on the concepts and methods of algorithmic randomness.

Algorithmic randomness, Brownian motion, and stochastic calculus

Bjørn Kjos-Hanssen.

During the years 2006-2009 I have worked with Nerode and with Szabados on algorithmic randomness and Brownian motion, following up on the founding work of Asarin and Pokrovskii from the 1980s. Simultaneously Fouche and his students have obtained many results in the area. Next in Fall 2012 I have taught a course on stochastic calculus to finance students, which brought up the question of the level of algorithmic randomness required for the calculation of quadratic variation and for Ito's lemma. I plan to present the old results and some preliminary thoughts on the new questions.

Infinite Games and Reverse Mathematics

Kazuyuki Tanaka.
Tohoku University, Japan.

Reverse Mathematics is an ongoing program of foundations of mathematics whose goal is to pin down exactly the logical or set-theoretical requirement for proving a theorem of ordinary mathematics. Most of the work on Reverse Mathematics has been carried out in subsystems of Z_2, or indeed $\Pi^1_2 - \mathrm{CA}_0$. However, the determinacy of infinite games is not tractable within those subsystems. In fact, Montalbán and Shore recently showed that $(\Sigma^0_3)_{<\omega}$ determinacy is not provable in Z_2. In this talk, we overview our long-range study on characterization of determinacy in terms of inductive definitions, transfinite recursion, transfinite induction, etc. In particular, we pin down the determinacy strength of Δ^0_3 games by several new types of inductive definitions.

Bayes optimal strategy for single item betting game

Kei Takeuchi.
Japan.

We discuss Bayesian strategies in unbounded forecasting game with quadratic hedge. By reducing the game to one-sided unbounded forecasting game without a hedge, we derive properties of Bayesian strategies in the game with quadratic hedge. Our results can be generalized to games with hedges other than the quadratic hedge and to multidimensional games.

Kuhn's theorem for natural strategies

Ryota Matsuo.
Nagoya university, graduate school of information science,
Japan.