ELC Seminar on Algorithmic Randomness

Hongo Campus, The University of Tokyo, Japan

13 - 14 May 2013

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Contents

1 General information

Title:
ELC Seminar on Algorithmic Randomness

Date:
13 - 14 May 2013

Venue:
237 Chemistry Building, Hongo Campus, The University of Tokyo, Japan.
Enter Science Building 7, go up one floor and then through the connecting corridor.
The details are on the website.

Local Organizers:
Akitoshi Kawamura (kawamura@is.s.u-tokyo.ac.jp),
Kenshi Miyabe (research@kenshi.miyabe.name)

This workshop is jointly sponsored by
"Exploring the Limits of Computation (ELC)"
and
"First, Aihara Innovative Mathematical Modelling Project".

2 Programme

Monday 13

Speaker 1. 9:30 – 12:30 Takayuki Kihara (JAIST, Japan)
Lowness for randomness and set theory of the real line.

12:30 – 14:00 *Lunch Break.*

Speaker 2. 14:00 – 17:00 Kenshi Miyabe (University of Tokyo, Japan)
Uniform relativization and almost uniform relativization.

Tuesday 14

Speaker 3. 10:00 – 11:30 Akitoshi Kawamura (University of Tokyo, Japan)
Polynomial-time randomness and differentiability.

3 Abstracts

Lowness for randomness and set theory of the real line.

Takayuki Kihara
JAIST, Japan.

If an infinite binary sequence is captured by a *small* subset of Cantor space, it shall be considered as far from being *random*. Roughly speaking, an infinite binary sequence is said to be *"effectively" random* if it is not captured by a *null* set with an "effective" Borel code. In the above definition, the word "null" shall be able to be replaced with any *ideal* in Cantor space. With this line, many researchers in algorithmic randomness theory have studied the notion of effective dimension: an infinite binary sequence is *effective Hausdorff dimension* $\leq d$ if for every $e > d$, it is captured by an $\mathcal{H}^e$-null set with a "effective" Borel code, where $\mathcal{H}^e$ denotes the e-dimensional Hausdorff measure.

In set theory, there is a depthful study on *σ-ideals* in Cantor space. Among them are the *strong measure zero* sets $\mathcal{SN}$, the *meager-additive* sets $\mathcal{M}^+$, and the *null-additive* sets $\mathcal{N}^+$. As proved by Laver in 1976, (under the consistency of ZFC) there is a model of ZFC with no uncountable set in $\mathcal{SN}$ (hence, in $\mathcal{M}^+$ and $\mathcal{N}^+$). Nevertheless we will see that their effective versions produce fruitful results on algorithmic randomness. More precisely, we characterize lowness for randomness by using the effective versions of these σ-ideals, and we justify the belief that the concept of Kolmogoriv complexity is a tool to estimate how difficult it is to capture an infinite sequence in a small set.

Uniform relativization and almost uniform relativization.

Kenshi Miyabe.
University of Tokyo, Japan.

The topic of this talk is van Lambalgen's theorem, which says that $A \oplus B$ is Martin-Löf random if and only if A is Martin-Löf random and B

is Martin-Löf random relative to B [4]. Such a theorem should hold for all natural randomness notions. Although some researchers found that van Lambalgen's theorem does not hold for some randomness notions, this is because the relativization is not appropriate.

In the first part, we review such motivation of introducing *uniform relativization* and show that van Lambalgen's theorem holds for uniform Schnorr randomness and uniformly computable randomness (in a weaker sense). This part is mainly from Miyabe [2] and Miyabe and Rute [3].

In the second part, we study van Lambalgen's theorem for uniform Kurtz randomness. A little unexpectedly, van Lambalgen's theorem does not hold for uniform Kurtz randomness. This part is from Kihara and Miyabe [1].

In the last part, we introduce another relativization, which we call *almost uniform relativization*. Actually we can show that van Lambalgen's theorem holds for almost uniform weak n-randomness for all $n \geq 1$. I also raise some questions relating to this problem.

References

[1] T. Kihara and K. Miyabe. Uniform Kurtz randomness. In preparation.

[2] K. Miyabe. Truth-table Schnorr randomness and truth-table reducible randomness. *Mathematical Logic Quarterly*, 57(3):323–338, 2011.

[3] K. Miyabe and J. Rute. Van Lambalgen's Theorem for uniformly relative Schnorr and computable randomness. To appear in Proceedings of the Twelfth Asian Logic Conference.

[4] M. van Lambalgen. *Random sequences*. PhD thesis, University of Amsterdam, 1987.

Polynomial-time randomness and differentiability.

Akitoshi Kawamura.
University of Tokyo, Japan.

The Brattka-Miller-Nies Theorem states that a real number is computably random if and only if every nondecreasing computable real function is differentiable at it. The proof is more involved than this simple statement might suggest, because we need to bridge between infinite sequences and the real numbers they represent, essentially proving the base invariance of randomness. In this talk I will look closely into this argument and discuss the polynomial-time (and other complexity) versions of the theorem and related statements. (Ongoing work with Kenshi Miyabe.)