

An introduction to algorithmic randomness

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10 Jan 2011



What is Randomness?



Randomness and probability



What is Randomness? [History of Randomness](#) [The relation with other notions](#)

- Is randomness artificial or natural?
- How about probability?
- How related are these?




How to use



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- The random killing of innocent people
- a random sample/selection
- The information is processed in a random order.
- He grabbed a pair of random jeans and an old red shirt.
- She dodged the random items that were on the concrete floor.
- The winning numbers are randomly selected by computer.



The definition in dictionaries



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- done without any regular pattern (by OALD)
- made without method or conscious decision (by ODE)
- (Statistics) governed by or involving equal chances for each item (by ODE)

The relation with probability

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No! - probability \Rightarrow randomness

Yes! - randomness \Rightarrow probability

History of Randomness

Before 1900

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The understandings of probability

- Aristotle & Christian Church
- Pascal - recursive determination
- Fermat - the enumeration of combinations (1654)
- Arnauld - the first person to use the word “probability”
- Bernoulli - subjective
- Laplace - equally possible
- 19C - empiricism, frequency

Around 1900

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- Hilbert's 6th problem - Axiomatize all of physics
- measure theory by Riemann, Jordan, Borel, Lebesgue, Radon
- Kolmogorov (1933)

Many critics by von Mises, de Finetti et al.

Kolmogorov 1963

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Kolmogorov 1963

I have already expressed the view that the basis for the applicability of the results of the mathematical theory of probability to real 'random phenomena' must depend on some form of the *frequency concept of probability*, the unavoidable nature of which has been established by von Mises in a spirited manner.

von Mises 1919 -

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First the Collective - then the probability

00100110010100111110...

The probability is the limit of the relative frequency of all (appropriate) subsequences.

- Church suggested the use of computability.
- Ville strengthened the notion by martingales.
- Wald studied it further.



Martin-Löf randomness 1966



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- Martin-Löf 1966 - typicalness
- Schnorr 1971 - unpredictability
- Levin 1973, Schnorr 1973, Chaitin 1975 - incompressibility
- van Lambalgen's Theorem 1987

Applications

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- Solomonoff induction
- Minimum Description Length, Minimum Message Length
- Game-theoretic probability by Vovk & Shafer



On a general space



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- computable analysis by Weihrauch 2000-
- randomness on a computable metric space by Gács 2005, Hoyrup & Rojas 2009-

The relation with other notions



Need modification



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- imprecise probability
- Dempster-Shafer Theory

Statistical test

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— Solomonoff 2011 —

Subjectivity in science has usually been regarded as Evil. - that it is something that does not occur in “true science” - that if it does occur, the results are not “science” at all. The great statistician, R. A. Fisher, was of this opinion. He wanted to make statistics “a true science” free of the subjectivity that had been so much a part of its history. I feel that Fisher was seriously wrong in this matter, and that his work in this area has profoundly damaged the understandings of statistics in the scientific community - damage from which it is recovering all too slowly.

Problem

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They believe that

Thesis

An event with small probability is unlikely to occur.

Then which event?

Prediction

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- Minimum Description Length, Minimum Message Length
- Defensive Forecasting

They can be interpreted as

Prediction

The best prediction searches the measure for which a sequence is random.

Ergodic theory

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Ergodic theorems

The “time average” is equal to the “space average” almost everywhere.

This can be interpreted as

Ergodic theorems

Unpredictability of the next place is based on unpredictability of the initial value.