Schnorr triviality is equivalent to being a basis for Schnorr randomness

Kenshi Miyabe*

We present some new characterizations of Schnorr triviality. The well-known notion of K-triviality is defined using complexity K via a prefix-free machine, with which Martin-Löf randomness has a characterization. In a similar manner, Schnorr triviality is defined using complexity via a computable measure machine, with which Schnorr randomness has a characterization. Further, we have a characterization of Schnorr randomness via decidable prefix-free machine. Hence, we should also have a characterization of Schnorr triviality using complexity via a decidable prefix-free machine.

Definition 1. *A set $A \in 2^\omega$ is called* weakly decidable prefix-free machine reducible *to a set $B \in 2^\omega$ (denoted by $A \leq_{\mathrm{wdm}} B$) if for each decidable prefix-free machine M and a computable order g, there exists a decidable prefix-free machine N such that*

$$(\exists d)(\forall n) K_N(A \upharpoonright n) \leq K_M(B \upharpoonright n) + g(n) + d.$$

It can be clearly observed that the relation $\leq_{\mathrm{wdm}}$ is reflexive and transitive. This reducibility has a strong connection with Schnorr randomness.

Theorem 2 (Bienvenu and Merkle [1])**.** *A set X is Schnorr random iff*

$$(\exists d)(\forall n) K_M(X \upharpoonright n) \geq n - g(n) - d$$

for all decidable prefix-free machines M and all computable orders g.

Theorem 3. *If a set A is Schnorr random and $A \leq_{\mathrm{wdm}} B$, then B is Schnorr random.*

Definition 4. *We say that a set A is* weakly trivial for decidable prefix-free machines *if A is weakly decidable prefix-free machine reducible to $\emptyset$.*

Theorem 5. *A set is Schnorr trivial iff it is weakly trivial for decidable prefix-free machines.*

It should be noted that numerous characterizations of Schnorr triviality have the following form: for any computable object, there exists another computable

*Research Institute for Mathematical Sciences, Kyoto University, kmiyabe@kurims.kyoto-u.ac.jp

object such that the real is in some object. By defining a basis for Schnorr randomness in a similar manner, we can show the equivalence to Schnorr triviality while Franklin and Stephan [2] showed that there exists a Schnorr trivial set that is not truth-table reducible to any Schnorr random set. Further, it should be noted that Franklin, Stephan and Yu [3] studied a base for Schnorr randomness, which is however a notion different from the one considered in this study.

At the same time, we also consider a basis for tt-reducible randomness. For the definition of tt-reducible randomness, refer to [4].

Definition 6. *Let d be a tt-reducible martingale. Then, a set X is A-tt-reducible random for d if*

$$(\exists d)(\forall n)d^A(X \restriction n) \leq d.$$

Further, a set X is A-tt-Schnorr random for d if for each computable order g,

$$(\exists d)(\forall n)d^A(X \restriction n) \leq h(n) + d.$$

Definition 7. *A set A is a basis fot tt-reducible randomness if, for each tt-reducible martingale d, there exists a set X such that $A \leq_{\mathrm{tt}} X$ and X is A-tt-reducible random for d.*

A set A is a basis fot tt-Schnorr randomness if, for each tt-reducible martingale d, there exists a set X such that $A \leq_{\mathrm{tt}} X$ and X is A-tt-Schnorr random for d.

Theorem 8. *The following are equivalent for a set A:*

(i) *A is Schnorr trivial,*

(ii) *A is a basis for tt-reducible randomness,*

(iii) *A is a basis for tt-Schnorr randomness.*

References

[1] L. Bienvenu and W. Merkle. Reconciling data compression and kolmogorov complexity. In L. Arge, C. Cachin, T. Jurdziński, and A. Tarlecki, editors, *Automata, Languages and Programming*, volume 4596 of *Lecture Notes in Computer Science*, pages 643–654, Berlin, 2007. Springer.

[2] J. N. Y. Franklin and F. Stephan. Schnorr trivial sets and truth-table reducibility. *Journal of Symbolic Logic*, 75(2):501–521, 2010.

[3] J. N. Y. Franklin, F. Stephan, and L. Yu. Relativizations of randomness and genericity notions. *Bulletin of the London Mathematical Society*, 43(4):721–733, 2011.

[4] K. Miyabe. Truth-table Schnorr randomness and truth-table reducible randomness. *Mathematical Logic Quarterly*, 57(3):323–338, 2011.