

- KENSHI MIYABE, *Variation of weakly computable reals in Solovay reducibility*.
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In computability theory, a real number α is termed *weakly computable* if there exists a computable sequence of rational numbers $(a_n)_n$ that converges to α and whose total variation, given by $|a_0| + \sum_{n \in \omega} |a_{n+1} - a_n|$, is finite. Miller (2017) defined a weakly computable real α as *variation nonrandom* if it possesses a computable approximation $(a_n)_n$ where the total variation is not Martin-Löf random. If no such approximation exists, α is called *variation random*.

This work investigates the relationship between a weakly computable real and its variation in the context of Solovay reducibility.

For any weakly computable real α , its variation β is always a left-c.e. real, and α is Solovay reducible to β ($\alpha \leq_S \beta$). Conversely, if β is a left-c.e. real such that $\alpha \leq_S \beta$, then we can find a computable approximation $(a_n)_n$ and a natural number q such that the variation of this sequence is $q\beta$. Consequently, determining the variation is equivalent to finding a left-c.e. real that is Solovay above α . This result further implies an algebraic characterization of the Solovay reducibility relation ($\alpha \leq_S \beta$) when α is weakly computable and β is left-c.e.

[1] JOSEPH S. MILLER, *On work of Barmapalias and Lewis-Pye: A derivation on the d.c.e. reals*, ***Computability and Complexity – Essays Dedicated to Rodney G. Downey on the Occasion of His 60th Birthday***, (Adam R. Day, Michael R. Fellows, Noam Greenberg, Bakhadyr Khoussainov, Alexander G. Melnikov and Frances A. Rosamond, editors), vol. 10010, Springer, 2017, pp. 644–659.